In conclusion, the authors wish to express their gratitude to S. I. Anisimov for his interest in this work and to O. A. Ponomarev and T. M. Martem'yanova for a useful discussion.

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# NEAR FIELD DIFFRACTED AT A DIELECTRIC WEDGE

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The integral equations of macroscopic dynamics [2] are used in [1] as the basis of a solution to the problem of the diffraction of a plane electromagnetic wave with a known polarization at a rectangular dielectric wedge. Expressions are given in this paper for the total electromagnetic field both inside a dielectric wedge of arbitrary flare angle and outside the wedge. The method used is the same as in [1].

## 1. Field Structure inside the Dielectric Wedge

Suppose that a plane electromagnetic wave is incident on a dielectric wedge of arbitrary flare angle $\alpha$. The dielectric permittivity $\varepsilon$ and the magnetic permeability $\mu$ of the wedge are in general complex and of arbitrary value. Without loss of generality, we can choose the polarization of the incident wave so that the field has the nonzero components

$$\mathbf{E}_0 = (E_{x0},\ 0,\ 0),\ \mathbf{H} = (0,\ H_{\rho 0},\ H_{\varphi 0}),$$

where

$$E_{x0}(\rho, \varphi) = E_{x0} e^{ik\rho\cos(\varphi-\varphi_0)};$$

$\varphi_0$ is the angle of incidence reckoned from the face $\varphi = 0$ (Fig. 1).

Then the field inside the wedge will have the same polarization and nonzero field components $\mathbf{E} = (E_x, 0, 0)$ and $\mathbf{H} = (0, H_\rho, H_\varphi)$, where $H_\rho$, $H_\varphi$ are cylindrical field components. The field can be represented as a set of plane refracted waves and an edge wave in the form of a Sommerfeld integral

$$E_x(\rho, \varphi) = \sum_j A_j e^{ik\rho\sqrt{\varepsilon\mu}\cos(\varphi-\psi_j)} + \int_{G_0} e^{ik\rho\sqrt{\varepsilon\mu}\cos(\varphi-\eta)} f(\eta)\, d\eta;$$

$$H_\varphi(\rho, \varphi) = -\sqrt{\frac{\varepsilon}{\mu}} \left\{ \sum_j A_j e^{ik\rho\sqrt{\varepsilon\mu}\cos(\varphi-\psi_j)} \cos(\psi_j - \varphi) + \int_{G_0} e^{ik\rho\sqrt{\varepsilon\mu}\cos(\varphi-\eta)} \cos(\eta - \varphi) \cdot f(\eta)\, d\eta \right\}; \quad (1.1)$$

$$H_\rho(\rho, \varphi) = \sqrt{\frac{\varepsilon}{\mu}} \left\{ \sum_j A_j e^{ik\rho\sqrt{\varepsilon\mu}\cos(\varphi-\psi_j)} \sin(\psi_j - \varphi) + \int_{G_0} e^{ik\rho\sqrt{\varepsilon\mu}\cos(\varphi-\eta)} \sin(\eta - \varphi) \cdot f(\eta)\, d\eta. \right.$$

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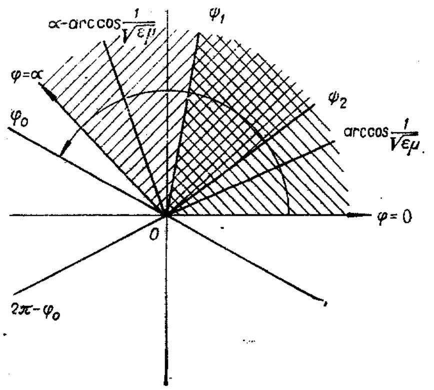


Fig. 1

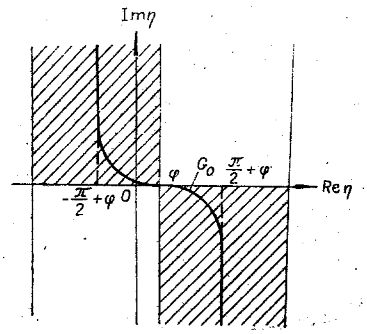


Fig. 2

Let us consider the first part of the solution – the set of plane refracted waves. Three situations arise, depending on how the faces of the wedge are illuminated by the incident wave. We take these cases separately.

If the incident wave illuminates only one face ($\varphi = 0$ or $\varphi = \alpha$), then only one refracted wave is excited inside the wedge (and this lies in the regions $0 \leq \varphi \leq \psi_1$ or $\psi_2 \leq \varphi \leq \alpha$, respectively). The wave amplitudes are defined by equations which correspond to the Fresnel equations: for illumination of the $\varphi = 0$ face

$$A_1 = \frac{2E_{0x} \sin \varphi_0 \cdot \cos \psi_1 \cdot \mu}{\sin (\varphi_0 + \psi_1) + (\mu - 1) \sin \varphi_0 \cdot \cos \psi_1}; \tag{1.2}$$

and for illumination of the $\varphi = \alpha$ face

$$A_2 = \frac{2E_{0x} \sin (\alpha - \varphi_0) \cdot \cos (\alpha - \psi_2) \cdot \mu}{\sin (2\alpha - \varphi_0 - \psi_2) + (\mu - 1) \sin (\alpha - \varphi_0) \cdot \cos (\alpha - \psi_2)}, \tag{1.3}$$

where the refraction angles $\psi_1$ and $\psi_2$, measured from the face $\varphi = 0$, can be found from the well-known laws for the refraction of electromagnetic waves at a boundary with a dielectric: with the given geometry we have

$$\sqrt{\varepsilon\mu} \cos \psi_1 = \cos \varphi_0, \quad \sqrt{\varepsilon\mu} \cos (\alpha - \psi_2) = \cos (\alpha - \varphi_0). \tag{1.4}$$

In the general case when both faces of the wedge are illuminated ($0 \leq \varphi_0 \leq \alpha$) we have two refracted waves whose amplitudes are given by (1.2) and (1.3). However, we must distinguish various cases.

Assume that the dielectric wedge is made from a material for which $\varepsilon\mu > 1$; then the following situations are possible.

1. If $\alpha < \pi/2$, then both refracted waves exist at all points inside the wedge. Thus for each of the internal points $r \in V$ the refracted plane wave is of the form

$$E_{\text{pl.int}} = \sum_{j=1}^{2} A_j e^{ik\rho\sqrt{\varepsilon\mu} \cos (\psi_j - \varphi)}, \tag{1.5}$$

where $A_{1,\,2}$ and $\psi_{1,\,2}$ are defined by (1.2)-(1.4).

2. If $\alpha > \pi/2$, then we can consider the inside of the wedge as consisting of three separate regions (see Fig. 1). In the first region there is only the one wave refracted at the face $\varphi = \alpha$, and the internal plane wave can be represented by the single term

$$E_{\text{pl}} = A_2 e^{ik\rho\sqrt{\varepsilon\mu} \cos (\psi_2 - \varphi)}, \quad \alpha > \varphi > \psi_1.$$

In the second region $\psi_1 > \varphi > \psi_2$ there are two refracted waves together and $E_{\text{pl}}$ is given by the full sum (1.5).

In the third region $\psi_2 > \varphi > 0$ there is again only the one refracted wave formed by the first face

$$E_{\text{pl}} = A_1 e^{ik\rho\sqrt{\varepsilon\mu} \cos (\psi_1 - \varphi)}, \quad \psi_2 > \varphi > 0.$$

If the dielectric wedge is made from a material which is less dense than the ambient medium then additional situations are possible and these need to be considered separately. In what follows we everywhere assume $\varepsilon\mu > 1$.

We now consider the remaining term in (1.1), which is in the form of a Sommerfeld-type integral with the weighting function $f(\eta)$. This term gives the edge wave which once again is defined by different relationships depending on the angle of observation $\varphi$ of the transmitted wave. Before turning to definite expressions for the weighting function, we consider the general nature of the diffraction solutions represented in the form of the Sommerfeld integral whose contour $G_0$ is shown in Fig. 2. Solutions in this form satisfy all the requirements applied to integral-type solutions of the Maxwell equations.

We can assume that they are finite, continuous, single-valued on the Riemann surface for all $\rho > 0$, have the required singularity at $\rho = 0$, and satisfy the principle of radiation at infinity [3].

It is not difficult to prove by direct calculations that the differential Maxwell equations

$$\begin{gathered} ik\mu H_\rho = (1/\rho)\partial E_x/\partial\varphi;\ ik\mu H_\varphi = -\partial E_x/\partial\rho; \\ -ik\varepsilon E_x = (1/\rho)\partial(\rho H_\varphi)/\partial\rho - (1/\rho)\partial H_\rho/\partial\varphi \end{gathered} \tag{1.6}$$

satisfy (1.1) independently of the form of the function $f(\eta)$.

We now write out the expressions which show how the weighting function depends on the angle of observation of the transmitted wave. We consider the general case where both faces of the wedge are illuminated. When $\alpha - \arccos(1/\sqrt{\varepsilon\mu}) < \varphi < \alpha$, $0 < \varphi < \arccos(1/\sqrt{\varepsilon\mu})$, $\mathrm{Re}\,\eta = \varphi$ the solutions can be represented as branched functions on a Riemann surface which is constructed above the plane of the complex variable $\eta(z)$ for the functions $z = \arccos(\sqrt{\varepsilon\mu}\cos\eta)$, $z = \alpha - \arccos[\sqrt{\varepsilon\mu}\cos(\alpha - \eta)]$:

$$f\left(\arccos\frac{\cos(u/n)}{\sqrt{\varepsilon\mu}}\right) = \frac{A_j}{2\pi i n}\,\frac{\sin\frac{\alpha}{n}\sqrt{\varepsilon\mu - \cos^2\frac{u}{n}}\cdot g_{sc}(u,\psi_j)}{\sqrt{\varepsilon\mu - \cos^2\frac{u}{n}} + \mu\sin\frac{u}{n}}; \tag{1.7}$$

$$f\left(\frac{\alpha}{n} - \arccos\frac{\cos\left(\frac{\alpha-u}{n}\right)}{\sqrt{\varepsilon\mu}}\right) = -\frac{A_j}{2\pi i n}\,\frac{\sin\frac{\alpha}{n}\sqrt{\varepsilon\mu - \cos^2\left(\frac{\alpha-u}{n}\right)}\cdot g_{sc}(u,\psi_j)}{\left[\sqrt{\varepsilon\mu - \cos^2\left(\frac{\alpha-u}{n}\right)} + \mu\sin\frac{u}{n}\right]}, \tag{1.8}$$

where

$$g_{sc}(u,\psi_j) = \frac{\left\{(\varepsilon-1)\mu + (\mu-1)\sqrt{\varepsilon\mu}\cos\left(\frac{\psi_j-u}{n}\right)\right\}}{\left(\sqrt{\varepsilon\mu}\cos\frac{\psi_j}{n} - \cos\frac{u}{n}\right)\left[\sqrt{\varepsilon\mu}\cos\left(\frac{\alpha-\psi_j}{n}\right) - \cos\left(\frac{\alpha-u}{n}\right)\right]}. \tag{1.9}$$

When $\arccos(1/\sqrt{\varepsilon\mu}) < \varphi < \psi_2$, $\psi_1 < \varphi < \alpha - \arccos(1/\sqrt{\varepsilon\mu})$ the branched solutions go over to the normal solutions corresponding to the case j = 1, 2

$$f\left(\arccos\frac{\cos u}{\sqrt{\varepsilon\mu}}\right) = \frac{A_j}{2\pi i}\,\frac{\sqrt{\varepsilon\mu - \cos^2 u}\cdot\sin\alpha\cdot g(u,\psi_j)}{(\sqrt{\varepsilon\mu - \cos^2 u} + \mu\sin u)}; \tag{1.10}$$

$$f\left(\alpha - \arccos\frac{\cos(\alpha-u)}{\sqrt{\varepsilon\mu}}\right) = -\frac{A_j}{2\pi i}\,\frac{\sqrt{\varepsilon\mu - \cos^2(\alpha-u)}\cdot g(u,\psi_j)\cdot\sin\alpha}{[\sqrt{\varepsilon\mu - \cos^2(\alpha-u)} + \mu\sin(\alpha-u)]}, \tag{1.11}$$

where

$$g(u,\psi_j) = \frac{(\varepsilon-1)\mu + (\mu-1)\sqrt{\varepsilon\mu}\cos(\psi_j-u)}{\left(\sqrt{\varepsilon\mu}\cos\frac{\psi_j}{n} - \cos u\right)\left[\sqrt{\varepsilon\mu}\cos(\alpha-\psi_j) - \cos(\alpha-u)\right]}. \tag{1.12}$$

Finally, when $\psi_2 < \varphi < \psi_1$, we get a solution which is the inverse to that for j = 1, 2

$$f\left(\arccos\frac{\cos u}{\sqrt{\varepsilon\mu}}\right) = \frac{\sin\alpha\sqrt{\varepsilon\mu - \cos^2 u}}{2\pi i(\sqrt{\varepsilon\mu - \cos^2 u} + \mu\sin u)}\sum_{j=1}^{2}(-1)^j A_j g(u,\psi_j)\times$$

$$\times \frac{\{2u-\alpha+\arccos[\sqrt{\varepsilon\mu}\cos(\alpha-\psi_j)]-\arccos[\sqrt{\varepsilon\mu}\cos\psi_j]\}}{\{\alpha-\arccos(\sqrt{\varepsilon\mu}\cos\psi_j)-\arccos[\sqrt{\varepsilon\mu}\cos(\alpha-\psi_j)]\}};$$

$$f\left(\alpha-\arccos\frac{\cos(\alpha-u)}{\sqrt{\varepsilon\mu}}\right)=-\frac{\sin\alpha\sqrt{\varepsilon\mu-\cos^2(\alpha-u)}}{2\pi i[\sqrt{\varepsilon\mu-\cos^2(\alpha-u)}+\mu\sin(\alpha-u)]}\times$$

$$\times\sum_{j=1}^{2}(-1)^j A_j g(u,\psi_j)\frac{\{2u-\alpha+\arccos[\sqrt{\varepsilon\mu}\cos(\alpha-\psi_j)]-\arccos(\sqrt{\varepsilon\mu}\cos\psi_j)\}}{\{\alpha-\arccos(\sqrt{\varepsilon\mu}\cos\psi_j)-\arccos[\sqrt{\varepsilon\mu}\cos(\alpha-\psi_j)]\}},$$

where $g(u, \psi_j)$ is defined by (1.12) and $A_j$ and $\psi_j$ by (1.2)-(1.4).

If only one face of the wedge is illuminated ($\varphi = 0$ or $\varphi = \alpha$), the weighting function is given as follows: for $0 < \varphi < \arccos(1/\sqrt{\varepsilon\mu})$, $\alpha - \arccos(1/\sqrt{\varepsilon\mu}) < \varphi < \alpha$, $f(\eta)$ is defined by (1.7) and (1.8); for $(1/\sqrt{\varepsilon\mu}) < \varphi < \alpha - \arccos(1/\sqrt{\varepsilon\mu})$, it is given by (1.10) and (1.11); j = 1 when the face $\varphi = 0$ is illuminated and j = 2 for illumination of the face $\varphi = \alpha$.

## 2. Structure of the Scattered Field on the Dielectric Wedge

The general form of the electromagnetic field outside the wedge [$\pi \geq \varphi_0 \geq \alpha$, $\alpha \leq 2\arccos(1/\sqrt{\varepsilon\mu})$] is

$$E_x(\rho,\varphi)=\begin{cases} E_{x0}(\rho,\varphi)-\dfrac{E_{x0}\left(\sqrt{\frac{\varepsilon}{\mu}}\sin\psi_1-\sin\varphi_0\right)e^{ik\rho\cos(\varphi+\varphi_0)}}{\left(\sqrt{\frac{\varepsilon}{\mu}}\sin\psi_1+\sin\varphi_0\right)}+P_{E_x}(\rho,\varphi), & 2\pi-\varphi_0\leqslant\varphi\leqslant 2\pi,\\ E_{x0}(\rho,\varphi)+P_{E_x}(\rho,\varphi), & \varphi_0\leqslant\varphi\leqslant 2\pi-\varphi_0,\\ P_{E_x}(\rho,\varphi), & \alpha\leqslant\varphi\leqslant\varphi_0;\end{cases} \quad (2.1)$$

$$H_\rho(\rho,\varphi)=\begin{cases} E_{x0}(\rho,\varphi)\sin(\varphi_0-\varphi)+\dfrac{E_{x0}\left(\sqrt{\frac{\varepsilon}{\mu}}\sin\psi_1-\sin\varphi_0\right)e^{ik\rho\cos(\varphi+\varphi_0)}}{\left(\sqrt{\frac{\varepsilon}{\mu}}\sin\psi_1+\sin\varphi_0\right)}\times\\ \quad\times\sin(\varphi+\varphi_0)+P_{H_\rho}(\rho,\varphi), & 2\pi-\varphi_0\leqslant\varphi\leqslant 2\pi,\\ E_{x0}(\rho,\varphi)\sin(\varphi_0-\varphi)+P_{H_\rho}(\rho,\varphi), & \varphi_0\leqslant\varphi\leqslant 2\pi-\varphi_0,\\ P_{H_\rho}(\rho,\varphi), & \alpha\leqslant\varphi\leqslant\varphi_0;\end{cases} \quad (2.2)$$

$$H_\varphi(\rho,\varphi)=\begin{cases} -E_{x0}(\rho,\varphi)\cos(\varphi-\varphi_0)+\dfrac{\cos(\varphi+\varphi_0)E_{x0}\left(\sqrt{\frac{\varepsilon}{\mu}}\sin\psi_1-\sin\varphi_0\right)e^{ik\rho\cos(\varphi+\varphi_0)}}{\left(\sqrt{\frac{\varepsilon}{\mu}}\sin\psi_1+\sin\varphi_0\right)}+P_{H_\varphi}(\rho,\varphi), & 2\pi-\varphi_0\leqslant\varphi\leqslant 2\pi,\\ -E_{x0}(\rho,\varphi)\cos(\varphi-\varphi_0)+P_{H_\varphi}(\rho,\varphi), & \varphi_0\leqslant\varphi\leqslant 2\pi-\varphi_0,\\ P_{H_\varphi}(\rho,\varphi), & \alpha\leqslant\varphi\leqslant\varphi_0,\end{cases} \quad (2.3)$$

where

$$P_{E_x}(\rho,\varphi)=-\frac{\sin\frac{\alpha}{n}}{4\pi i n\mu}\left\{\int_{G_1}e^{ik\rho\cos(u+\varphi)}g(u)c(u)\,du-\int_{G_2}e^{ik\rho\cos(u-\varphi)}g(u)\,du\right\},\quad \varphi\in(\pi,2\pi);$$

$$P_{E_x}(\rho,\varphi)=-\frac{\sin\frac{\alpha}{n}}{4\pi i n\mu}\left\{\int_{G_1}e^{ik\rho\cos(u-\varphi)}g(u)\,du-\int_{G_2}e^{ik\rho\cos(u+\varphi-2\alpha)}g(u)c(\alpha-u)\,du\right\},\quad\varphi\in(\alpha,\pi); \quad (2.4)$$

$$P_{H_\rho}(\rho,\varphi)=\frac{\sin\frac{\alpha}{n}}{4\pi i n\mu}\left\{\int_{G_1}e^{ik\rho\cos(u+\varphi)}g(u)\sin(u+\varphi)\cdot c(u)\,du+\int_{G_2}e^{ik\rho\cos(u+\varphi)}g(u)\sin(u-\varphi)\,du\right\},\quad\varphi\in(\pi,2\pi);$$

$$P_{H_\rho}(\rho,\varphi) = -\frac{\sin\frac{\alpha}{n}}{4\pi i n\mu}\left\{\int_{G_1} e^{ik\rho\cos(u-\varphi)} g(u)\sin(u-\varphi)\,du + \right.$$

$$\left. + \int_{G_2} e^{ik\rho\cos(u+\varphi-2\alpha)} g(u)\sin(u+\varphi-2\alpha)\cdot c(\alpha-u)\,du\right\}, \quad \varphi\in(\pi,\alpha);$$

$$P_{H_\varphi}(\rho,\varphi) = -\frac{\sin\frac{\alpha}{n}}{4\pi i n\mu}\left\{-\int_{G_1} e^{ik\rho\cos(u+\varphi)} g(u)\cos(u+\varphi)\cdot c(u)\,du + \right.$$

$$\left. + \int_{G_2} e^{ik\rho\cos(u-\varphi)} g(u)\cos(u-\varphi)\,du\right\}, \quad \varphi\in(\pi,2\pi);$$

$$P_{H_\varphi}(\rho,\varphi) = -\frac{\sin\frac{\alpha}{n}}{4\pi i n\mu}\left\{-\int_{G_1} e^{ik\rho\cos(u-\varphi)} g(u)\cos(u-\varphi)\,du + \right.$$

$$\left. + \int_{G_2} e^{ik\rho\cos(u+\varphi-2\alpha)} g(u)\cos(u+\varphi-2\alpha)\cdot c(\alpha-u)\,du\right\}, \quad \varphi\in(\pi,\alpha);$$

g(u) is given by (1.9),

$$c(u) = \frac{\sqrt{\varepsilon\mu-\cos^2\frac{u}{n}} - \mu\sin\frac{u}{n}}{\sqrt{\varepsilon\mu-\cos^2\frac{u}{n}} + \mu\sin\frac{u}{n}},$$

and the contours $G_1$ and $G_2$ are defined, respectively, by the equations of the curves:

$$u = \arccos(\sqrt{\varepsilon\mu}\cos\eta), \quad \eta\in G_0;$$
$$u = \alpha - \arccos[\sqrt{\varepsilon\mu}\cos(\alpha-\eta)], \quad \eta\in G_0.$$

It can be shown that the scattered field components (2.1)-(2.3), which have been derived from the integral Maxwell equations [2], also satisfy the differential Maxwell equations (1.6).

## 3. Field Structure in the Neighborhood of the Wedge Faces

For the fields inside and outside the wedge we already have explicit expressions which satisfy the Maxwell equations and so we can immediately proceed to a study of the field structure in the neighborhood of the faces and to a simultaneous verification of the correctness and validity of the whole method of constructing the solution to the diffraction problem. We have to check that the solutions satisfy the boundary conditions on the side faces of the wedge:

$$E_{t_{sc}} = E_{t_{in}},\quad H_{t_{sc}} = H_{t_{in}},\quad D_{n_{sc}} = D_{n_{in}},\quad B_{n_{sc}} = B_{n_{in}} \quad \text{for } \varphi = 0,\quad \varphi=\alpha, \text{ where } E_t = E_x,\ H_t = H_\rho,\ B_n = B_\varphi.$$

In order to save space, we give detailed calculations for two cases only: we derive the expression for $E_x$ in the neighborhood of the face $\varphi = 0$ and at the same time we check that the tangential component of the electric field is continuous at $\varphi = 0$ and $\varphi = \alpha$. We make the change of variable $\eta - \varphi = u$ in the integral term of (1.1) and we expand the weighting function under the integral sign into a Taylor series in the small parameter $\varphi$. Limiting ourselves to the first two terms in the expansion and substituting the actual values of A, $\psi$, f[arccos(cos (u/n)/ $\sqrt{\varepsilon\mu}$)] from (1.1), (1.4), and (1.7), we get the following expression for the internal field for small $\varphi$:

$$E_x(\rho,0) = \frac{E_{x0}e^{ik\rho\cos\varphi_0}2\sin\varphi_0}{\left(\sqrt{\frac{\varepsilon}{\mu}}\sin\psi_1+\sin\varphi_0\right)} + \frac{\sin\frac{\alpha}{n}}{2\pi i n}\int_{G_1}\frac{e^{ik\rho\cos u}g(u)\sin\frac{u}{n}\,du}{\sqrt{\varepsilon\mu-\cos^2\frac{u}{n}}+\mu\sin\frac{u}{n}} + \varphi I(\rho);\quad I(\rho) = \int_{\eta\in G_0-\varphi} e^{ik\rho\sqrt{\varepsilon\mu}\cos\eta} f'_\varphi(\eta)\,d\eta. \quad (3.1)$$

The first expression in (2.1) for the $E_x$ component of the scattered field enables a solution to be constructed on the face $\varphi = 0$, where $P_{E_x}(\rho, \varphi)$ is given by the first equation in (2.4). In order to get the boundary value of the field we carry out the following transformations. We reduce the integrals to a single contour $G_1$ because the integrand is analytic over the internal region $D_1^+$ (defined by the contour $\Gamma = G_1 + G_2$). After some simple algebra we find that the value of the scattered field for small $\varphi$ is also given by (3.1), where

$$I(\rho) = -\frac{\sin\frac{\alpha}{n}}{4\pi i n\mu}\left\{\int\limits_{(u+\varphi)\in G_1} e^{ik\rho\cos u}\,[g(u)\,c(u)]'_{\varphi}\,du - \int\limits_{(u-\varphi)\in G_2} e^{ik\rho\cos u} g'_{\varphi}(u)\,du\right..$$

It can be seen from (3.1) that the internal and scattered fields are identical on the face $\varphi = 0$.

A similar test for the continuity of the tangential component of the electric field can be made on the boundary $\varphi = \alpha$ except that (1.8) is now used for the weighting function in the internal field and the contour $G_1$ is deformed into the contour $G_2$ in the integral term of the scattered field, for which the last equation in (2.1) enables the solution to be constructed on $\varphi = \alpha$. The boundary values of the internal and scattered fields on the face $\varphi = \alpha$ are then identical ($\psi < \alpha$):

$$E_x(\rho, \alpha) = -\frac{\sin\frac{\alpha}{n}}{2\pi i n}\int\limits_{G_2}\frac{e^{ik\rho\cos(u-\alpha)} g_p(u)\sin\left(\frac{\alpha-u}{n}\right)du}{\left[\sqrt{\varepsilon\mu - \cos^2\left(\frac{\alpha-u}{n}\right)} + \mu\sin\left(\frac{\alpha-u}{n}\right)\right]}.$$

If we carry out similar operations for the remaining field components (1.1), (2.2), and (2.3) we see that

$$H_{t_{sc}} = H_{t_{in}},\ B_{n_{sc}} = B_{n_{in}}\ \text{for}\quad \varphi = 0,\ \varphi = \alpha.$$

For the case where there are no re-reflections, i.e., large flare angles and small angles of incidence as measured from the surface normal, we can distinguish the following regions in the internal and external fields for illumination of the $\varphi = 0$ face. In the shadow region of the scattered field where $\alpha < \varphi < \varphi_0$ there is only the edge wave (for $k\rho\sqrt{\varepsilon\mu} \gg 1$ it is a cylindrical wave) and for $2\pi - \varphi_0 > \varphi > \varphi_0$ there is the incident and the edge wave. Thus the ray $\varphi = \varphi_0$ is the boundary between the illuminated region and the shadow region $\varphi_0 > \varphi > \alpha$. Similarly, the ray $\varphi = 2\pi - \varphi_0$ is the boundary between the region where there is a reflected wave ($2\pi - \varphi_0 < \varphi < 2\pi$) and the region where there is no reflected wave ($\varphi_0 < \varphi < 2\pi - \varphi_0$). In the internal wedge-shaped region, the shadow boundary $\varphi = \psi_1$ divides the physical space into two sections: $0 \le \varphi \le \psi_1$ and $\psi_1 \le \varphi \le \alpha$. In the first of these the cylindrical wave interferes with the plane refracted wave. In the geometrical shadow region ($\psi_1 \le \varphi \le \alpha$) the cylindrical wave is isolated from the other waves. Near the shadow boundary $\varphi = \psi_1, \varphi_0, 2\pi - \varphi_0$, i.e., in the penumbra zone, the field is more complex and cannot in general be expressed in terms of plane and cylindrical functions. A more detailed study of the edge wave shows that in the neighborhood of $\varphi = \psi_1$ the field is equal to

$$-\frac{e^{i(k\rho\sqrt{\varepsilon\mu}-\pi/4)-ik\rho\sqrt{\varepsilon\mu}s_0^2}\,2\sqrt{2\pi}\,if(\varphi)\,[\cos(\psi-\varphi)-1]}{s_0}\int\limits_{\infty s_0}^{\sqrt{2k\sqrt{\varepsilon\mu}\rho}\cdot s_0} e^{ig^2}dg, \qquad (3.2)$$

$$s_0^2 = 2\sin^2\left(\frac{\psi-\varphi}{2}\right),$$

with $f(\varphi)$ given by (1.11). The integral which appears in (3.2) is the Fresnel integral; its lower limit is equal to infinity in absolute value and its sign is taken from that of $\sin[(\psi - \varphi)/2]$. Thus the lower limit changes sign across the plane-wave boundary $\varphi = \psi$ and the integral undergoes a discontinuous jump which ensures that the diffraction field is continuous across the shadow boundary.

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